Impact of Correlated Log-Normal Shadowing on Two-Way Network Coded Cooperative Wireless Networks

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***Abstract*—In this letter, we provide a novel theoretical framework for studying the effects of correlated shadowing, in the number of relays that are capable of helping two nodes (sources) to exchange their messages. The relays use network coding to simultaneously transmit the received messages to the sources. We prove theoretically and verify by means of simulations that the average number of relays that are capable of forwarding the network coded message, is independent from any correlation between the links from one source to the relays. Finally, we apply this framework, to compute the network outage probability. The presented results are essential for the theoretical study of medium access control and relay selection protocols designed for network coded cooperative communications.**

***Index Terms*—Network coding, outage probability, two-way relay channel, shadowing, correlation.**

I. Introduction

BIDIRECTIONAL cooperative networks have attracted great research interest in recent years. Network coding (NC) [1] has been proposed as an efficient routing mechanism that enables the intermediate nodes in the network to process the incoming data before forwarding them to their final destinations. Even though the incorporation of NC in bidirectional cooperative networks leads to significant capacity improvement, it requires new schemes to optimize the medium access coordination of the relays such us: i) relay selection protocols [2]–[5], ii) medium access control (MAC) protocols [6]. Most of these schemes are evaluated by taking into account only the fast fading effects [2]–[5]. However, geographically proximate radio links experience correlated shadowing [7]. Therefore, shadowing is a crucial impairment that should be also considered.

An essential metric for studying the performance of cooperative communication protocols operating over fading conditions is the value of the probability of having exactly K relay to have decoded correctly the received data. In [eqs. (6),(7)][8] and [eqs. (2),(3)][9] the authors evaluate the performance of selection cooperation schemes by computing the value of the aforementioned probability, under fast fading conditions. The authors in [10], evaluated the average throughput of a MAC protocol designed for one way cooperative communications after studying the effects of uncorrelated shadowing to the average number of relays that receive packets of a given Quality-of-Service (QoS) requirement [eqs. (3),(5)][10]. This paper is a first attempt to study the effects of both fast fading and correlated shadowing in the number of relays that are suitable to participate in NC Cooperative (NCC) MAC and relay selection protocols designed for two way networks. The contribution of this paper is given as follows: i) we provide a novel theoretical framework for studying the effects of fast fading and correlated shadowing in the number of relays (active) that receive packets of a given QoS from both sources that act as destination as well ii) we prove theoretically that the average number of active relays is independent of the correlation between the links and iii) we apply this framework in order to estimate the network outage probability, that can be used for the investigation of single relay selection schemes for NCC communications.

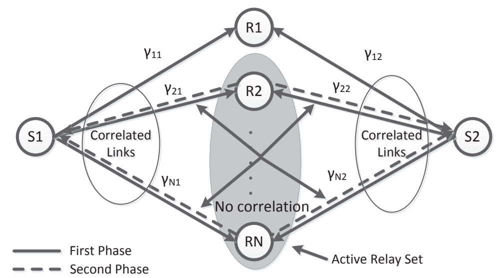

Fig. 1. *Two way cooperative network with N regenerative relays*

II. System Model

We consider a network with 2 sources (S_1, S_2) and N relays (R_i) as shown in Fig. 1. All the nodes are equipped with single-input-single-output transceivers designed for point to point communications. We assume that there is no direct link between $S_1 \rightarrow S_2$. Communication is performed in 2 phases: i) In phase 1: S_1, S_2 broadcast packets A, B, respectively, in an orthogonal way to R_i. The set of relays that accept packets from both sources (*active relay set* C_N), perform digital NC in the network layer[1] (i.e A xor B or a linear combination of A,B). ii) In phase 2: one of the active relays, determined by a coordination protocol, broadcasts the NC packet to S_1, S_2.

After taking into account both fast fading and correlated shadowing, we may write the instantaneous received power at R_i, $i = 1, \ldots, N$ from S_j, $j = 1, 2$ as $P_{R_{ij}} =$

[1]NC can be applied in cases where the individual packets contain errors [3]. However, the condition that the individual packets A,B, need to be accepted by the MAC in order to perform NC at the network layer, allows the use of SISO transcievers, without making any modifications in their PHY or MAC layer.

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$P_{T_j} \left|h_{f_{ij}}\right|^2 \left|h_{s_{ij}}\right|^2$ [10], where: i) P_{T_j} is the transmit-power from any source $j = 1, 2$ ii) $h_{f_{ij}}$ is the fast-fading coefficient, modeled as a Nakagami-m random variable (RV) with $E\left[\left|h_{f_{ij}}\right|^2\right] = 1$ iii) and $h_{s_{ij}}$ is the shadow-fading coefficient. We assume that fast fading ergodicity allows the calculation of $E\left[\left|h_{f_{ij}}\right|^2\right]$ from a sufficiently long sample realization (i.e. duration of a packet), and that shadowing, is a slowly varying procedure [7] that can be considered constant for both communication phases. With $\gamma_{ij} = P_{T_j} \left|h_{s_{ij}}\right|^2 E\left[\left|h_{f_{ij}}\right|^2\right]$, we denote the RV that corresponds to the average received power at the relay R_i from S_j, computed over the duration of a packet. Many experimental results have shown that γ_{ij} can be modeled as log-normal RVs. In decibels, $\gamma_{ij_{dB}} = 10log_{10}\left(\gamma_{ij}\right)$ is a normally distributed RV with mean value $\mu_{ij_{dB}}$ and variance $\sigma_{ij_{dB}}$. In order to take into account any possible correlation between $\gamma_{i1_{dB}}$ or $\gamma_{i2_{dB}}$ we may use the approach given in [11, eq. (8.1.6)], and write:

$$\gamma_{i1_{dB}} = \sigma_{i1_{dB}} \left(\sqrt{1-\lambda_{i1}^2} X_i + \lambda_{i1} X_0\right) + \mu_{i2_{dB}} \tag{1}$$

$$\gamma_{i2_{dB}} = \sigma_{i2_{dB}} \left(\sqrt{1-\lambda_{i2}^2} Y_i + \lambda_{i2} Y_0\right) + \mu_{i2_{dB}} \tag{2}$$

where $X_i, Y_i, X_0, Y_0 \sim \mathcal{N}(0,1)$ are i.i.d. normal RVs and $\lambda_{ij} \in (0,1)$ are scalar values that determine the amount of correlation between $\gamma_{ij_{dB}}$: $\rho\left(\gamma_{i1_{dB}}, \gamma_{j1_{dB}}\right) = E\left[\left(\gamma_{i1_{dB}} - \mu_{i1_{dB}}\right)\left(\gamma_{j1_{dB}} - \mu_{j1_{dB}}\right)\right] / \sigma_{i1_{dB}}\sigma_{j1_{dB}} = \lambda_{i1}\lambda_{j1}$, $\rho\left(\gamma_{i2_{dB}}, \gamma_{j2_{dB}}\right) = \lambda_{i2}\lambda_{j2}$ and $\rho\left(\gamma_{i1_{dB}}, \gamma_{j2_{dB}}\right) = 0$, $\forall i, j$, $i \neq j$.

Now let us focus on the metrics that verify the correct packet reception. In environments where shadowing is not considered, the ergodicity of fast fading allows the utilization of average metrics, such as the average symbol error probability (ASEP), to determine the acceptance of a packet. On the other hand, when shadowing is considered, the ASEP is a function of the shadowing coefficient (i.e., $ASEP_{ij} = f(E\left[\left|h_{f_{ij}}\right|^2\right], h_{s_{ij}})$), and the QoS requirement $ASEP_{ij} < p^*$ turns out to be equivalent to $\gamma_{ij} > \gamma_0$ [12]. Therefore R_i accepts a packet from S_j if $\gamma_{ij_{dB}} > \gamma_{0_{dB}}$[2].

III. Theoretical Framework

In this section we study the impact of the aforementioned physical layer parameters on the size of the active relay set $|C_N|$. In order to describe which one of the N relays have accepted packets transmitted by S_1, we introduce the notation b_{i1_N}, that corresponds to the N-bit binary representation of integer $i \in \left[0, 2^N - 1\right]$ (e.g. $b_{11_N} = 00\ldots01$). The position of 1s in the binary word b_{i1_N} indicates the relays in which the average received power $\gamma_{i1_{dB}}$ is above the threshold $\gamma_{0_{dB}}$. The probability of occurence for each one of the possible events b_{i1_N} $\forall i$, is denoted by $\Pr\left(b_{i1_N}, \gamma_0\right)$ i.e.,

$$\begin{aligned}
&\Pr\left(b_{11_N}, \gamma_0\right) = \Pr\left\{\gamma_{11_{dB}} \leq \gamma_{0_{dB}}, \ldots, \gamma_{N1_{dB}} > \gamma_{0_{dB}}\right\} \\
&= \int_{-\infty}^{\infty} \Pr\left\{\gamma_{11_{dB}} \leq \gamma_{0_{dB}}, \ldots, \gamma_{N1_{dB}} > \gamma_{0_{dB}} \left| X0 = t\right.\right\} f_{X_0}(t)\, dt \\
&= \int_{-\infty}^{\infty} \Pr\left\{X_1 \leq a1(t)\right\} \cdots \Pr\left\{X_N > aN(t)\right\} f_{X_0}(t)\, dt
\end{aligned} \tag{3}$$

where $a_{i1}(t) = \left(\gamma_{0_{dB}} - \mu_{i1_{dB}} - \sigma_{i1_{dB}}\lambda_{i1} t\right) / \left(\sigma_{i1_{dB}}\sqrt{1-\lambda_{i1}^2}\right)$ and $f_{X_0}(t)$ is the probability density function of X_0. Therefore the individual probabilities $\Pr\left(b_{i1_N}, \gamma_0\right)$ may be defined as:

$$\Pr\left(b_{i1_N}, \gamma_0\right) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \prod_{k=1}^{N} F_k\left(a_{k1}(t)\right) e^{\frac{-t^2}{2}}\, dt \tag{4}$$

$$F_k\left(a_{k1}(t)\right) = \begin{cases} Q\left(a_{k1}(t)\right), \ if \ b_{i1_N}(k) = 1 \\ 1 - Q\left(a_{k1}(t)\right), \ if \ b_{i1_N}(k) = 0 \end{cases}, k = 1, \ldots, N. \tag{5}$$

$Q(\cdot)$ is the standard one dimensional Gaussian Q-function. The single integral expression in eq. (4) can be evaluated by using the Gauss's formula with arbitrary interval:

$$\Pr\left(b_{i1_N}, \gamma_0\right) = \sum_{i=1}^{N_{GQR}} \frac{w_i}{\sqrt{\pi}} \prod_{k=1}^{N} F_k\left(a_{k1}\left(\sqrt{2} r_i\right)\right), \tag{6}$$

where w_i and r_i are the weights and roots of Gauss integration [13] and N_{GQR} is the degree of the Hermite polynomial. In a similar way, we may define the individual probabilities $\Pr\left(b_{i2_N}, \gamma_0\right)$, $i = 0, 1, \ldots 2^N - 1$.

The binary words b_{i1_N}, b_{j2_N} indicate which relays have accepted packets transmitted by S_1 and S_2 respectively. The active relay set in a network with N relays may be written as $\mathcal{C}_N = \{k \in [1, N] : b_{i1_N}(k) = 1, b_{j2_N}(k) = 1\}$. The probability that K relays have successfully received packets from both sources ($\Pr\{|\mathcal{C}_N| = K\}$) may be estimated after taking into account all the possible binary codewords b_{i1_N}, b_{j2_N} that satisfy $H_w\left(b_{i1_N} \odot b_{j2_N}\right) = K$. With $\odot$ we denote the bit wise AND operation, and $H_w(b)$ is the hamming weight function that returns the number of 1s in the binary word b. Thereafter:

$$\Pr\{|\mathcal{C}_N| = K\} = \sum_{i=0}^{2^N-1} \sum_{j \in A_i} \Pr\left(b_{i1_N}, \gamma_0\right) \Pr\left(b_{j2_N}, \gamma_0\right) \tag{7}$$

where $A_i = \left\{j \in \left[0, 2^N - 1\right] : H_w\left(b_{i1_N} \odot b_{j2_N}\right) = K\right\}$. The average size of the decoding set may be then computed as:

$$E\left[|\mathcal{C}_N|\right] = \sum_{i=1}^{N} i \Pr\{|\mathcal{C}_N| = i\} \tag{8}$$

IV. Applications

A. Impact of correlation on the average size of $\mathcal{C}_N$

In this section we prove that $E\left[|\mathcal{C}_N|\right]$ is independent of the correlation coefficients $\rho\left(\gamma_{i1_{dB}}, \gamma_{j1_{dB}}\right), \rho\left(\gamma_{i2_{dB}}, \gamma_{j2_{dB}}\right)$ $\forall i, j$. We initially study this metric for a network with 2 Relays. After selecting $\lambda_{11} = \lambda_{21} = \sqrt{\rho_1}$ and $\lambda_{12} = \lambda_{22} = \sqrt{\rho_2}$ the correlation between the RVs $\gamma_{i1_{dB}}$, for $i = 1, 2$ may be written as: $\rho\left(\gamma_{11_{dB}}, \gamma_{21_{dB}}\right) = \rho_1$ and $\rho\left(\gamma_{12_{dB}}, \gamma_{22_{dB}}\right) = \rho_2$. By applying eq. (7) for $N = 2$, we may write $\Pr\{|\mathcal{C}_2| = K\}$ for $K = 1, 2$ as:

$$\Pr\{|\boldsymbol{C}_2| = 2\} = \Pr\left(b_{31_2}, \gamma_0\right) \Pr\left(b_{32_2}, \gamma_0\right) \tag{9}$$

$$\begin{aligned}
\Pr\{|\boldsymbol{C}_2| = 1\} &= \Pr\left(b_{11_2}, \gamma_0\right) \left(\Pr\left(b_{12_2}, \gamma_0\right) + \Pr\left(b_{32_2}, \gamma_0\right)\right) \\
&+ \Pr\left(b_{21_2}, \gamma_0\right) \sum_{j=2}^{3} \Pr\left(b_{j2_2}, \gamma_0\right) + \Pr\left(b_{31_2}, \gamma_0\right) \sum_{j=1}^{2} \Pr\left(b_{j2_2}, \gamma_0\right)
\end{aligned} \tag{10}$$

where the probabilities $\Pr\left(b_{ij_2}, \gamma_0\right)$ may be computed from eq. (4) by setting $N = 2$.

[2] The threshold value γ_0 is given in closed form [10, eq. (1)] as a function of a target ASEP when assuming M-PSK or M-QAM modulation.

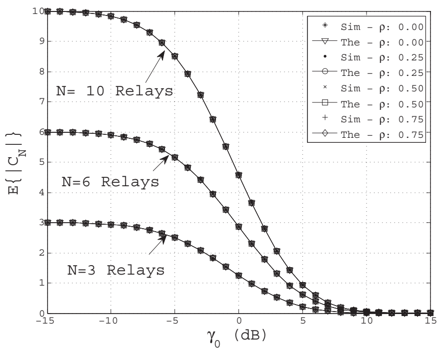

Fig. 2. Verification of proposition 1. $\mu_{i_{dB}} \sim \mathcal{U}(0,3)$,$\lambda_{ij} \sim \mathcal{U}(0,\rho)$

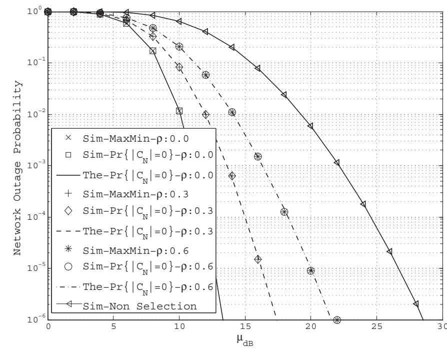

Fig. 3. Verification of proposition 2. $\gamma_{0_{dB}} = 9, \sigma_{dB} = 4,\ N = 10$

Lemma 1. *For any given* $0 \leq \rho < 1$, $\gamma_{0_{dB}}$, μ_{dB}, σ_{dB}, *we have*

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} Q\left(\frac{\gamma_{0_{dB}} - \mu_{dB} - \sigma_{dB}\sqrt{\rho}t}{\sigma_{dB}\sqrt{1-\rho}}\right) e^{-t^2/2} dt = Q\left(\frac{\gamma_{0_{dB}} - \mu_{dB}}{\sigma_{dB}}\right)$$

Proof: The proof is given in the appendix. ■

By employing lemma 1, eq. (8) is simplified to:

$$E\left[|\mathcal{C}_2|\right] = \sum_{i=1}^{2} Q\left(\frac{\gamma_{0_{dB}} - \mu_{i1_{dB}}}{\sigma_{i1_{dB}}}\right) Q\left(\frac{\gamma_{0_{dB}} - \mu_{i2_{dB}}}{\sigma_{i2_{dB}}}\right) \quad (11)$$

The generalization of this result, may be stated as follows:

Proposition 1. *In the network of Fig. 1, the average number of relays that accept packets from both sources, is independent of the correlation between the links and is given by:*

$$E\left[|\mathcal{C}_N|\right] = \sum_{i=1}^{N} Q\left(\frac{\gamma_{0_{dB}} - \mu_{i1_{dB}}}{\sigma_{i1_{dB}}}\right) Q\left(\frac{\gamma_{0_{dB}} - \mu_{i2_{dB}}}{\sigma_{i2_{dB}}}\right) \quad (12)$$

Proof: The proof is given in the appendix. ■

The authors in [eqs. (3),(5)][10], employed the average number of relays that have correctly decoded packets transmitted by a single source in order to compute the average throughput of a cooperative MAC. Similarly, the metric presented in eq. (12) may be used to compute the average throughput of MAC schemes designed for NCC communications (i.e. [6]).

B. Network Outage Probability

In this section we compute the probability of having no active relays in the network, assuming equal correlation between the links and identical statistics[3]. Under the aforementioned assumptions we derived the following proposition:

Proposition 2. *Assume the network of Fig. 1. Suppose that (a) the shadowing correlation between any pair of* $R_i \to S_1$ *link is equal to* ρ_1*, (b) the shadowing correlation of any pair of* $R_i \to S_2$ *links is equal to* ρ_2*, (c) all the links have the same statistics. Then the probability that none of the relays successfully receives packets from both sources, is given by*

$$\Pr\left\{|\mathcal{C}_N| = 0\right\} = \sum_{i=0}^{N} \binom{N}{i} \sum_{j=0}^{N-i} \binom{N-i}{j} P_{i1_N} P_{j2_N}, \quad (13)$$

$$P_{iz_N} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} Q\left(a_z(t)\right)^i Q\left(-a_z(t)\right)^{N-i} e^{-t^2/2} dt \quad (14)$$

$a_z(t) = \frac{\gamma_{0_{dB}} - \mu_{z_dB} - \sigma_{z_dB}\sqrt{\rho_z}t}{\sigma_{z_dB}\sqrt{1-\rho_z}}$, $z = 1, 2$. *This probability corresponds to the outage probability of the max-min relay selection [3], that selects for cooperation a single relay that satisfy the following condition* $r^* = max_i\left(min\left(\gamma_{i1_{dB}}, \gamma_{i2_{dB}}\right)\right)$.

Proof: The proof is given in the appendix. ■

This probability can be used as a reference outage probability for the investigation of relay selection schemes.

V. Numerical Results

In order to validate propositions 1 and 2, we conducted Monte Carlo simulations, which showed a very good agreement with the analytical results derived in Section IV. In Fig. 2 we provide simulation results for the average number of active relays in networks with different number of relays ($N = 3, 6, 10$). We consider a random correlation model where the parameters $\lambda_{i1}, \lambda_{i2}$ are randomly selected in the interval $[0, 1)$. For all the cases, the mean value of the links $\mu_{ij_{dB}}$ is randomly selected in the interval $[0, 3]\,dB$ while the variance is kept the same $\sigma_{ij_{dB}} = 7\ \forall i, j$. It is clearly shown that $E\left[|\mathcal{C}_N|\right]$ is independent of the correlation between the links.

Finally, Fig. 3 provides a) the network outage probability and b) the outage probability of max min relay selection scheme, for different correlation factors versus the mean $\mu_{i1_{dB}} = \mu_{i2_{dB}} = \mu_{dB}$ of the average received power $\gamma_{ij_{dB}}$ in a network with $N = 10$ relays. The single integral expressions were evaluated by using the Gauss Hermite method, where the value of $N_{GQR} = 100$. As a reference protocol we use the non selection scheme, where a fixed relay node (R_1) assists the bidirectional transmission, and therefore, the relaying does not benefit from the clustered relay structure. It is shown that i) outage probability curves (a), (b) coincide and ii) as correlation increases, the outage probabilities (a), (b) approaches the outage of the non selection scheme.

VI. Conclusions

In this paper, we provided a novel theoretical framework for studying the effects of fast fading and correlated shadowing, in the number of relays that are capable of participating in medium coordination schemes designed for NCC communications. We proved theoretically that correlation does not affect the average number of relays that accept packets from both sources. Finally, we computed the network outage probability assuming equal correlation between the links.

[3]Equal correlation between the identically ditributed links occurs by selecting $\lambda_{i1} = \sqrt{\rho_1}, \lambda_{i2} = \sqrt{\rho_2}$ and $\mu_{ij_{dB}} = \mu_{j_{dB}}$, $\sigma_{ij_{dB}} = \sigma_{j_{dB}} \forall i \in [1, N]\,,\ j = 1, 2$.

VII. APPENDIX

A. Proof of Lemma 1

Any RV $Z \sim N(\mu, \sigma^2)$ may be written in the form

$$Z = \sigma\sqrt{\rho}X_1 + \sigma\sqrt{1-\rho}X_2 + \mu \tag{15}$$

for any given $0 \le \rho < 1$, where $X_1, X_2 \sim N(0,1)$. Let $W = Z - \mu$, $X = \sigma\sqrt{\rho}X_1$, $Y = \sigma\sqrt{1-\rho}X_2$ then the above equation may be written as $W = X + Y$, where $W \sim N\left(0, \sigma^2\right)$, $X \sim N\left(0, \sigma^2\rho\right)$, $Y \sim N\left(0, \sigma^2\left(1-\rho\right)\right)$. The CDF of W can be written as $F_w(t) = \int_{-\infty}^{+\infty} F_y\left(t-x\right) f_x(x)dx$. After making some changes in variables the aforementioned equation may be written as follows:

$$F_z(t) = \frac{1}{\sqrt{2\pi\sigma\rho}} \int_{-\infty}^{+\infty} F_{x_2}\left(\frac{t-x-\mu}{\sigma\sqrt{1-\rho}}\right) e^{-\frac{x^2}{2\sigma^2\rho}} dx \Rightarrow$$

$$Q\left(\frac{t-\mu}{\sigma}\right) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} Q\left(\frac{t-\sigma\sqrt{\rho}x_1-\mu}{\sigma\sqrt{1-\rho}}\right) e^{-\frac{x_1^2}{2}} dx_1$$

B. Proof of Proposition 1

We proved that $E\left[|\mathcal{C}_2|\right]$, is given by eq. (11). Let us assume that for a K relay network, we have $E\left[|\mathcal{C}_K|\right] = \sum_{i=1}^{K} Q\left(a_{i1}\right) Q\left(a_{i2}\right)$, where $a_{ij} = \left(\gamma_{0_{dB}} - \mu_{ij_{dB}}\right)/\sigma_{ij_{dB}}$. Now we will prove that:

$$E\left[|\mathcal{C}_{K+1}|\right] = E\left[|\mathcal{C}_K|\right] + Q\left(a_{(K+1)1}\right) Q\left(a_{(K+1)2}\right) \tag{16}$$

To simplify notations we set $P_{R_i}^K = \Pr\left\{|\boldsymbol{C}_K| = i\right\}$ and we assume that $B_{K+1} = \left\{\gamma_{(K+1)1} > \gamma_0, \gamma_{(K+1)2} > \gamma_0\right\}$ is the event that the R_{K+1} relay is active. Then we may write:

$$P_{R_i}^{K+1} = P_{R_i|\overline{B_{K+1}}}^K \Pr\left\{\overline{B_{K+1}}\right\} + P_{R_{i-1}|B_{K+1}}^K \Pr\left\{B_{K+1}\right\}$$

for $i = 1, \ldots, K$ and $P_{R_{K+1}}^{K+1} = P_{R_K|B_{K+1}}^K \Pr\left\{B_{K+1}\right\}$. Then we can write $E\left[|\mathcal{C}_{K+1}|\right]$ as:

$$\begin{aligned} E\left[|\mathcal{C}_{K+1}|\right] &= \sum_{i=1}^{K+1} i P_{R_i}^{K+1} \\ &= \sum_{i=1}^{K} i \left[P_{R_i|\overline{B_{K+1}}}^K \Pr\left\{\overline{B_{K+1}}\right\} + P_{R_i|B_{K+1}}^K \Pr\left\{B_{K+1}\right\}\right] \\ &+ \sum_{i=0}^{K} P_{R_i|B_{K+1}}^K \Pr\left\{B_{K+1}\right\} = E\left[|C_K|\right] + \Pr\left\{B_{K+1}\right\} \end{aligned} \tag{17}$$

where $\Pr\left\{B_{K+1}\right\} = Q\left(a_{(K+1)1}\right) Q\left(a_{(K+1)2}\right)$.

C. Proof of Proposition 2

The probabilitites $\Pr\left(b_{ij_N}, \gamma_0\right)$, after assuming equal correlation between the identically ditributed links, do not depend on the position of 1s but only on $H_w\left(b_{ij_N}\right)$, (e.g. $\Pr\left(b_{1j_N}, \gamma_0\right) = \Pr\left(b_{2^z j_N}, \gamma_0\right) = P_{1j_N}$, $z = 1, \ldots, N-1$). Therefore eq. (4) simplifies to eq. (14). Suppose that b_{w1_N}, b_{z2_N} are the binary codewords that denote which of the $S_1 \to R_i$, $S_2 \to R_i$ links are active. Given that $H_w\left(b_{w1_N}\right) = i, H_w\left(b_{z2_N}\right) = j \le N - i$ then we can select in total $\binom{N}{i}$ different b_{w1_N} binary words with $H_w\left(b_{w1_N}\right) = i$ each one with probability of occurrence P_{i1_N}. For each one of those b_{w1_N} binary words, there are $\binom{N-i}{j}$ binary words b_{z2_N} with $H_w\left(b_{z2_N}\right) = j$ such that $H_w\left(b_{w1_N} \odot b_{z2_N}\right) = 0$ and thus: $\Pr\left\{|\mathcal{C}_N| = 0 \mid H_w\left(b_{w1_N}\right) = i, H_w\left(b_{z2_N}\right) = j\right\} = \binom{N}{i} P_{i1_N} \binom{N-i}{j} P_{j2_N}$. If we take all those combinations into account for $i, j \in [0, N]$ with $j \le N - i$ we end up at eq. (13).

The outage probability $Pr\left\{max_i\left(min\left(\gamma_{i1_{dB}}, \gamma_{i2_{dB}}\right)\right) < \gamma_0\right\}$ may be written as $Pr\left\{\cap_{i=1}^{N} min\left(\gamma_{i1_{dB}}, \gamma_{i2_{dB}}\right) < \gamma_0\right\}$. Note that the condition $min\left(\gamma_{i1_{dB}}, \gamma_{i2_{dB}}\right) < \gamma_0$ is equivalent with the one that takes into account all the pairs of binary words b_{w1_N}, b_{z2_N} with $w, z \in \left[0, 2^N - 1\right]$ that results in $b_{w1_N}(i) \odot b_{z2_N}(i) = 0 \ \forall i \in [1, N]$. Therefore the above probability is equivalent with the probability of the union of $\left\{b_{w1_N}, b_{z2_N}\right\}$ that satisfy the condition $H_w\left(b_{w1_N} \odot b_{z2_N}\right) = 0$ and as a result corresponds to $\Pr\left\{|\mathcal{C}_N| = 0\right\}$.

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